MATH 1A:

LINEAR APPROXIMATION

$$f(x) \approx f(a) + f'(a)(x-a) \quad \text{FOR } x \approx a$$

TO APPROXIMATE $\sin \frac{1}{2}$

$$f(x) = \sin x \qquad 0 \approx \frac{1}{2} \quad \text{USE } a = 0$$

$$f(\tfrac{1}{2}) \approx f(0) + f'(0)(\tfrac{1}{2} - 0) \qquad f'(x) = \cos x$$

$$\sin \frac{1}{2} \approx \sin 0 + (\cos 0)\frac{1}{2}$$

$$\sin \frac{1}{2} \approx 0 + 1 \cdot \frac{1}{2} = \frac{1}{2} \leftarrow$$ NOT CLOSE TO CALCULATOR 0.4755...

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

IF $x = \frac{1}{2}$

$$x - \frac{x^3}{3!} \approx 0.4791666\ldots$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} \approx 0.47947270833$$

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \approx 0.4794255332$$

CALCULATOR

$\sin \frac{1}{2} \approx 0.4794255386$

HOW TO FIND THE "INFINITE POLYNOMIAL"

FOR A FUNCTION

1. HOW TO FIND THE COEFFICIENTS

2. WHICH VALUES OF X CAN WE SUBSTITUTE IN + HAVE THE "POLYNOMIAL" BE A GOOD ESTIMATE

3. HOW MANY TERMS DO WE NEED TO GET A GOOD ESTIMATE

11.1

SEQUENCES

A SEQUENCE IS A LIST OF NUMBERS IN WHICH ORDER IS RELEVANT
(NOT NECESSARILY SORTED)

eg. 1, 3, 5, 7, 9, 11

1, 4, 0, 8, 8, 6, 4, 8, 2, 6, 8

OR

$\{1, 3, 5, 7, 9, 11, \ldots\}$ ← MATH IC NOTATION

$\{a_1, a_2, a_3, a_4, a_5, \ldots\}$

$\{a_n\}$ ← IMPLIED n STARTS AT 1

$\{a_n\}_{n=1}^{\infty}$ ← EXPLICITLY STATED n STARTS AT 1

$$\left\{ \frac{2n+3}{n!} \right\}_{n=1}^{\infty}$$

$$= \left\{ \frac{2\cdot 1+3}{1!}, \frac{2\cdot 2+3}{2!}, \frac{2\cdot 3+3}{3!}, \ldots \right\}$$

$$= \left\{ 5, \frac{7}{2}, \frac{3}{2}, \ldots \right\}$$

$$\left\{ \frac{5}{9}, -\frac{6}{16}, \frac{7}{25}, -\frac{8}{36}, \ldots \right\}$$

$n=1$ $n=2$ $n=3$ $n=4$

$$\left\{ (-1)^0 \frac{(1+4)}{(1+2)^2}, (-1)^1 \frac{(2+4)}{(2+2)^2}, (-1)^2 \frac{(3+4)}{(3+2)^2}, (-1)^3 \frac{(4+4)}{(4+2)^2}, \ldots \right\}$$

$$\left\{ (-1)^{n-1} \frac{n+4}{(n+2)^2} \right\}_{n=1}^{\infty}$$